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TRANSFORMATION OF THE ELECTRIC FIELD AT PROPAGATION OF ACOUSTIC WAVES AND THEIR TRANSITION INTO PLASMA WAVES

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SUMMARY

This paper considers the question of excitation theory of electric fields during a nearly vertical upward propagation of an acoustic wave at altitudes corresponding to that of the F_2-layer of the ionosphere. The occurring charge division conditions the transformation of the electric field in the form of a circularly polarized wave. The transformation of the electric field in height is investigated in such a wave.

A system of plasma equations is proposed, taking into account the neutrals, whose motion is given in the form of a propagating viscous acoustic wave. The conclusions of theory are utilized for the interpretation of regularities linked with the peculiarity in the propagation of electric fields. An estimate is given of the effectiveness of inhomogeneity formation, taking into account the dependence of electron and ion concentration with height.

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One of the important questions of physics of the ionosphere is that of the influence of atmospheric motions of acoustic, gravitational or Lamb type waves on the excitation and propagation of plasma oscillations. The development of large-scale inhomogeneities of electron concentration, called moving inhomogeneities, was the object of consideration in a series of works [1--5].

Apparently, the appearance of such type of disturbances is linked with the excitation inside the ionosphere of inner gravitational waves. The propagation of inner waves is analyzed on the basis of standard hydrodynamics equations, when the entire multi-component medium is investigated as a single unit. With such an approach one may single out directly only the character of motion of the neutral component, which, as far as the density is concerned, is prevailing at the levels of E- and F-layers and below. However, when observing moving inhomogeneities with the aid of radio methods, one may practically fix the variations in the distribution of electron concentration. From the above it is clear that a substantiated comparison of theory with observations can be made only on the basis of the solution of the problem of "excitation" of electron and ion inhomogeneities during the propagation of acoustic or gravitational waves. When resolving this problem, one should take into account, besides the interaction of electrons and ions with neutral particles, the Earth's magnetic field and the contribution of ambipolar diffusion and even some other effects.

Variations of Electron and Ion Concentration in the Ionosphere at Given Motion of Neutral Particles. We shall formulate the basic equations determining the spatial and temporal variations of electron and ion concentrations in large-scale inhomogeneities of the ionosphere (in E- and F-regions) [5 - 8].

We shall start from the quasihydrodynamic equations of motion for electrons and ions

$$\frac{\partial \mathbf{v}_e}{\partial t} + \gamma_{e0}(\mathbf{v}_e - \mathbf{v}_0) = -\frac{e}{m}\left\{\mathbf{E} + \frac{1}{c}[\mathbf{v}_e\mathbf{H}]\right\},$$

$$\frac{\partial \mathbf{v}_i}{\partial t} + \gamma_{i0}(\mathbf{v}_i - \mathbf{v}_0) = \frac{e}{M}\left\{\mathbf{E} + \frac{1}{c}[\mathbf{v}_i\mathbf{H}]\right\} \qquad (1)$$

and continuity equations. Here v_e and v_i are respectively the coordinated velocities of electons and ions; (for simplicity, we assume that there is only one kind of ions); $\vec{H}$ is the intensity of the magnetic field of the Earth; $\vec{E}$ is the strength of the electric field, hindering the onset of a noncompensated charge in the plasma; $\underline{c}$ is the speed of light in the vacuum; $\underline{e}$ is the absolute value of the charge of the electron. The quantities γ_{e0} γ_{i0} characterize respectively the effective collision frequencies of electrons and ions with molecules, whose velocity is v_0. Absent in Eqs.(1) are the terms taking into account the electron and ion viscosities, which are immaterial in the analysis of large-scale inhomogeneities. The collisions between charged particles are not taken into account either, which is well founded so long as the frequency γ_{e0} is much greater than the collision frequencies of electrons with ions; this condition is fulfilled below the F-layer maximum. The terms containing the velocity components of molecules play the role of an external force, stimulating the appearance of inhomogeneities of electron concentration.

In the Maxwellian equations we neglect the displacement currents

$$\mathrm{rot}\,\mathbf{H} = \frac{4\pi e}{c}\{N_i(z)\,\mathbf{v}_i - N_e(z)\,\mathbf{v}_e\},\quad \mathrm{rot}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} \tag{2}$$

considering the magnetic permeability and the permittivity (dielectric constant) as being equal to the unity.

<u>On the Transformation of the Electric Field at Propagation and Transformation of Acoustic Waves into Plasma Waves</u>. Let us consider the nearly vertical propagation of a viscous acoustic wave in an isotropic isothermic atmosphere. The atmosphere constitutes a gaseous half-space $z > 0$ above a hard plane in a uniform gravitational field ($q_z = -q$). If we direct the axis $\underline{z}$ perpendicularly to the boundary plane, the equilibrium distribution of density will be written in the form

$$p = p_0 e^{-z/z_0},\quad \rho = \rho_0 e^{-z/z_0},\quad z_0 = \frac{a_0^2}{\gamma g}.$$

The analytical solution of this problem is well known for great heights ($z/z_0 \gg 1$) [9, 10]

$$v_{ac}(z) = v_{0z} \approx v_0 \exp\left\{\frac{u_0^2}{2\nu_0\omega_n^0}\left(i - 2\frac{\delta n}{\omega_n^0}\right)e^{-z/z_0}\right\}. \tag{3}$$

The acoustic wave has a component along the axis $\underline{x}$, but $v_{0x} \ll v_{0z}$, so that

$$v_{0x} = v_{0z}\sin\alpha,\qquad |\alpha| \ll 1.$$

We assume that at $z = z_1$ there is a sharp atmosphere-ionosphere interface whereupon for $z \geqslant z_1$ the conductivity becomes substantial ($\sigma \gg 1$), so that we can neglect the displacements currents in the Maxwellian equations. Along the axis $\underline{z}$ the magnetic field is constant. The variation with height of electron and ion concentration is assumed to be exponential [11], and the value of collision frequencies of electrons γ_{e0} and ions γ_{i0} with neutrals — independent of the coordinate z.

We shall consider small two-dimensional oscillations of the ionosphere along the axes x and y. In the entire space $z > z_1$ the problem will be described by a system of equations for collective oscillations of plasma (1) and electromagnetic field (2). On the strength of the above assumptions $\partial/\partial t = i\omega$, $\partial/\partial x = \partial/\partial y = 0$ are fulfilled; then, for monochromatic oscillations the system of Eqs.(1) for perturbed values of $\vec{v}_i$, $\vec{v}_e$, $\vec{E}$, $\vec{h}$ will take the form

$$i\omega v_{ix} + \gamma_{i0}(v_{ix} - v_{0x}) = \frac{e}{M}E_x - \omega_{iH} v_{iy},$$

$$i\omega v_{iy} + \gamma_{i0}(v_{iy} - v_{iy}) = \frac{e}{M}E_y - \omega_{iH} v_{ix},$$

$$i\omega v_{ex} + \gamma_{e0}(v_{ex} - v_{0x}) = -\frac{e}{m}E_x - \omega_{eH} v_{ey}, \tag{4}$$

$$i\omega v_{ey} + \gamma_{e0}(v_{ey} - v_{0y}) = -\frac{e}{m}E_x + \omega_{eH} v_{ex}.$$

The Maxwellian equations

$$\frac{\partial h_x}{\partial z} = \frac{4\pi e}{c}N(z)(v_{iy} - v_{ey}), \quad \frac{\partial h_y}{\partial z} = -\frac{4\pi e}{c}N(z)(v_{ix} - v_{ex}),$$

$$\frac{\partial E_x}{\partial z} = -i\frac{\omega}{c}h_y, \quad \frac{\partial E_y}{\partial z} = i\frac{\omega}{c}h_x. \tag{5}$$

The cyclotron frequencies of ions and electrons are respectively denoted by ω_{iH} and ω_{eH}; at the same time in (5) $N_e = N_i = N(z)$,

$$\omega_{iH} = \frac{eH_0}{Mc}, \quad \omega_{ee} = \frac{eH_0}{mc},$$

$$v_{ix} = r_i(\omega)\left[(i\omega + \gamma_{i0})\left(\gamma_{i0}v_{0x} + \frac{e}{M}E_x\right) + \omega_{iH}\left(\gamma_{i0}v_{0y} + \frac{e}{M}E_y\right)\right],$$

$$v_{iy} = r_i(\omega)\left[-\omega_{iH}\left(\gamma_{i0}v_{0x} + \frac{e}{M}E_x\right) + p_i(\omega)\cdot\omega\left(\gamma_{0i}v_{0y} + \frac{e}{M}E_y\right)\right],$$

$$v_{ex} = r_e(\omega)\left[(i\omega + \gamma_{e0})\left(\gamma_{e0}v_{0x} - \frac{e}{m}E_x\right) - \omega_{eH}\left(\gamma_{e0}v_{0y} + \frac{e}{m}E_y\right)\right],$$

$$v_{0y} = r_e(\omega)\left[\omega_{eH}\left(\gamma_{e0}v_{0x} + \frac{e}{m}E_x\right) - p_e(\omega)\cdot\omega\left(\gamma_{e0}v_{0y} + \frac{e}{m}E_y\right)\right],$$

where

$$r_e(\omega) = \frac{1}{\omega_{eH}^2 - \omega^2 + 2i\omega\gamma_{e0}}, \quad r_i(\omega) = \frac{1}{\omega_{iH}^2 - \omega^2 + 2i\omega\gamma_{i0}},$$

$$p_e(\omega) = \frac{\gamma_{e0}\omega + i(\omega^2 + 2\gamma_{e0}^2)}{\omega^2 + \gamma_{e0}^2}, \quad p_i(\omega) = \frac{\gamma_{i0}\omega + i(\omega^2 + 2\gamma_{i0}^2)}{\omega^2 + \gamma_{i0}^2}$$

Omitting the intermediate operations, we obtain the equations for the determination of the electric field

$$\frac{d^2E_x}{dz^2} = \frac{4\pi e}{c^2} N(z) i\omega (v_{ix} - v_{ex}),$$

$$\frac{d^2E_y}{dz^2} = \frac{4\pi e}{c^2} N(z) i\omega (v_{iy} - v_{ey}).$$

Substituting the values $v_{ix}, v_{ex}, v_{iy}, v_{ey}$, we obtain

$$\frac{d^2E_x}{dz^2} = \frac{4\pi N(z) e}{c^2} i\omega \Big\{ [r_i(\omega)\gamma_{i0}(i\omega + \gamma_{i0}) - r_e(\omega)\gamma_{e0}(i\omega + \gamma_{e0})]\, v_{0x} +$$
$$+ \frac{e}{M}\left[r_i(\omega)(i\omega + \gamma_{i0}) + \frac{M}{m} r_e(\omega)(i\omega + \gamma_{e0}) \right] E_x + \frac{e}{M}\left[r_i(\omega)\omega_{iH} + \frac{M}{m} r_e(\omega)\omega_{eH} \right] E_y +$$
$$+ [r_i(\omega)\gamma_{i0}\omega_{iH} + r_e(\omega)\gamma_{e0}\omega_{eH}]\, v_{0y}.$$
$$\frac{d^2E_y}{dz^2} = \frac{4\pi N(z) e}{c^2} i\omega \Big\{ -[r_i(\omega)\gamma_{i0}\omega_{iH} + r_e(\omega)\gamma_{e0}\omega_{eH}]\, v_{0x} -$$
$$- \frac{e}{M}\left[r_i(\omega)\omega_{iH} + \frac{M}{m} r_e(\omega)\omega_{eH} \right] E_x + \frac{e\omega}{M}\left[r_i(\omega)p_i(\omega) + \frac{M}{m} r_e(\omega)p_e(\omega) \right] E_y + \tag{6}$$
$$+ \omega\,[p_i(\omega)\gamma_{i0} + p_e(\omega)\gamma_{e0}]\, v_{0y} \Big\}.$$

We shall consider Eqs.(6) only in their application to processes in the E- and F-layers. The inequality $\gamma_{i0} \ll \omega_{iH}$ is well satisfied at these heights, and from it follows immediately the limitation $\gamma_{e0} \ll \omega_{eH}$. Estimates show [8] that even for large-scale inhomogeneities, without speaking of small-scale processes, the addends in the equations of motion, proportional to vector components of terrestrial acceleration, may be dropped, so that Eqs.(1) are correct with great precision.

Propagation of a Circularly-Polarized Electric Wave. We shall consider the question of propagation of harmonic oscillations with frequency ω, when the conditions $\gamma_{i0} \ll \omega$, $\gamma_{e0} \ll \omega$, $\omega < \omega_{iH}$, $\omega < \omega_{eH}$ are satisfied.

Then the system of Eqs.(6) is significantly simplified; neglecting the quadratic terms of the form $(\gamma/\omega)^2$, we shall find the values of the coefficients at v_{0x}, v_{0y}, E_x and E_y

$$a_0 \approx i\omega[r_i(\omega)\gamma_{i0} - r_e(\omega)\gamma_{e0}],$$
$$b_0 \approx r_i(\omega)\gamma_{i0}\omega_{iH} + r_e(\omega)\gamma_{e0}\omega_{eH},$$
$$a_1 \approx i\omega \frac{e}{M}\left[r_i(\omega) + \frac{M}{m} r_e(\omega) \right],$$
$$b_1 \approx \frac{e}{M}\left[r_i(\omega)\omega_{iH} + \frac{M}{m} r_e(\omega)\omega_{eH} \right]. \tag{7}$$

Then, if we postulate $v_{0y} = iv_{0x}$, $E_y = iE_x$. Eqs.(6) for E_x and E_y will be identities. Therefore, Eqs.(6) determine the propagation of the circularly polarized wave $E = E_x \pm iE_y$, $v = v_{0x} \mp iv_{0y}$

$$\frac{d^2E_x}{dz^2} = \frac{4\pi e}{c^2} N(z) i\omega \{(a_0 + ib_0) v_{0x} + (a_1 + ib_1) E_x\}.$$

Computing the combinations $a_0 + ib_0$ and $a_1 + ib_1$, we obtain

$$\frac{d^2E}{dz^2} + \frac{\omega^2}{c^2}\left[\frac{r_i(\omega)}{\omega}\Omega^2(\omega_{iH} + \omega) + \frac{r_e(\omega)}{\omega}\omega_p^2(\omega_{eH} + \omega)\right] e^{-z/h} E +$$
$$+ \frac{\omega^2\Omega^2}{c^2 e}[r_i(\omega)\gamma_i(\omega_{iH} + \omega) r_e(\omega)\gamma_e(\omega_{eH} - \omega)] e^{-z/h} v_{0x} = 0. \qquad (8)$$

We denoted by ω_p the Langmuir frequency of plasma's natural oscillations ($\omega_p = 4\pi e^2 N_0/m$); Ω is the natural frequency of oscillations of ions ($\Omega^2 = 4\pi e^2 N_0/M$), and the distribution of electron density with height is given in the exponential form $N = N_0 \exp(-z/h)$, where $\underline{h}$ is the height of uniform ionosphere, $E = E_x$.

As follows from (8), the appearance of electron density inhomogeneities, and with it, that of the electric field, are linked with the variation of the longitudinal component of molecule velocity in the direction of the geomagnetic field $\vec{H}$. Some preliminary results may be obtained without resolving (8) from the analysis of the expression containing v_{0x}. This addend plays in (8) the role of a "constraining force", conditioning the transformation of the electric field. Without touching upon the question of formation and propagation of moving inhomogeneities of electron concentration [8], we shall attempt to find the solution of (8). Certain complications arise on the strength of dependences of γ_i and γ_e on height. Following is the approximation that may be used for the number of collisions:

$$\gamma_i = \gamma_{i0} e^{-z/h}, \qquad \gamma_e = \gamma_{e0} e^{-z/h}.$$

We neglect the certain difference in the values of $\underline{h}$ for γ_i and γ_e.

Transforming the denominators of the expressions for the coefficients E and v_{0x}, making use of the smallness condition $\beta = \gamma/\omega \ll 1$, neglecting the terms proportional to β^2 and denoting $z/h = \xi$, we obtain

$$\frac{d^2E}{d\xi^2} + \frac{\omega^2 h^2}{c^2}\left[a - i(b_1 + b_2)e^{-\xi}\right]e^{-2\xi}E + \frac{\omega^2\Omega^2 h^2}{c^2 e}(c_1 + c_2)e^{-2\xi}v_{0x} = 0,$$

$$a = -\frac{\Omega^2}{\omega(\omega_{iH} - \omega)} + \frac{\omega_p^2}{\omega(\omega_{eH} - \omega)},$$

$$b_1 = -\frac{2\Omega^2}{\omega_{iH}(\omega_{iH} - \omega)\left(1 - \frac{\omega^2}{\omega_{iH}}\right)}\beta_i, \quad b_2 = 2\frac{\omega_p^2}{\omega_{eH}^2}\left(\frac{\omega}{\omega_{eH}}\right)\frac{1 + \frac{\omega_{eH}}{\omega}}{\left(1 - \frac{\omega^2}{\omega_{eH}^2}\right)^2}\beta_e,$$

$$c_1 = -\frac{1}{\left(1 - \frac{\omega}{\omega_{iH}}\right)}\beta_i, \quad c_2 = \frac{1}{\left(1 + \frac{\omega}{\omega_{eH}}\right)}\beta_e, \quad \beta_i = \frac{\gamma_{i0}}{\omega_{iH}}, \quad \beta_e = \frac{\gamma_{e0}}{\omega_{eH}}.$$

Let us introduce new variables and new denotations

$$E(x) = E(\xi), \quad v_{0x} = v(x), \quad x = e^{-\xi},$$

$$A = a\frac{\omega^2 h^2}{c^2}, \quad C = -\frac{\omega^2\Omega^2 h^2}{c^2 e}\left[-\frac{1}{1 - \frac{\omega}{\omega_{iH}}} + \frac{\gamma_{e0}}{\gamma_{i0}}\left(\frac{1}{1 + \frac{\omega}{\omega_{eH}}}\right)\frac{\omega_{iH}}{\omega_{eH}}\right],$$

$$\beta = \beta_i.$$

$$\Gamma = -\frac{2\omega^2 h^2}{c^2}\left[\frac{\Omega^2}{\omega_{iH}(\omega_{iH} - \omega)\left(1 - \frac{\omega^2}{\omega_{iH}^2}\right)} + \frac{\omega_p^2}{\omega_{eH}(\omega_{eH} - \omega)\left(1 - \frac{\omega^2}{\omega_{eH}}\right)}\left(\frac{\gamma_{e0}\,\omega_{iH}}{\gamma_{i0}\,\omega_{eH}}\right)\right]$$

After the respective transformations, the equation for E assumes the form

$$xE'' + E' + (A - i\beta\Gamma x)E + c\beta x v = 0. \qquad (10)$$

After some transformations, the uniform Eq,(10) may be reduced to an equation, parent to Whittaker's [10], of which the solution is expressed in the final resort by a degenerated hypergeometrical function ${}_1F_1(\frac{1}{2} + m - k, 1 + 2m, \rho x)$. In order to find a partial solution of the inhomogenous Eq.(10), it is necessary to compute the Wronskian, and then the integral of such complex and cumbersome expressions, that we shall be again compelled to resort to limiting cases: to search for ${}_1F_1$ and ${}_1F_1{}^2$ in the form of expansion in series by powers of smallness. Utilizing the Pachhammer functions' expansion, we see that difficulty in computing integrals remains, so that it is not possible to compute them analytically. Note that in order to find analytical solutions the physically reasonable solutions of the problem may be obtained only within the limits of the F_2-layer, where the values of collision frequencies γ_e and γ_i with height may vary by a factor of 2 to 3. We shall consider that within the indicated limits, γ_i and γ_e do not vary. Their constancy means that we artificially increase the damping of the electric field with height. In the approximation γ_i = const, γ_e = const, Eq.(10) is significantly simplified

$$xE'' + E' + (A - i\beta\Gamma)E + \beta C v = 0 \qquad (11)$$

and is related to the type of Bessel's inhomogenous equation [10]. The basic solutions of the homogenous equation have the form

$$E_1 = I_0(2\sqrt{A - i\beta\Gamma}\, x^{1/2}), \qquad E_2 = N_0(2\sqrt{A - i\beta\Gamma}\, x^{1/2}).$$

As is well known, in case of cylindrical functions, the Wronskian is

$$W\left[I_0(2\sqrt{A - i\beta\Gamma}\, x^{1/2}),\ N_0(2\sqrt{A - i\beta\Gamma}\, x^{1/2})\right] = \frac{1}{\pi x^{1/2}}.$$

As follows from the theory of differential equations, the solution of Eq.(11) may be represented in the form

$$E = c_1 E_1 + c_2 E_2 + \beta C E_2 \int \frac{E_1 v(x)}{W}\, dx - \beta C E_1 \int \frac{E_2 v(x)}{W}\, dx.$$

In our case

$$E = c_1 I_0((2\varepsilon\sqrt{x}) + c_2 N_0(2\varepsilon\sqrt{x}) + \pi\beta C N_0(2\varepsilon\sqrt{x}) \int \sqrt{x}\, I_0(2\varepsilon\sqrt{x})\, v(x)\, dx -$$
$$- \pi\beta C I_0(2\varepsilon\sqrt{x}) \int \sqrt{x}\, N_0(2\varepsilon\sqrt{x})\, v(x)\, dx,$$

where $\varepsilon = \sqrt{A - i\beta\Gamma}$. According to formula (3)

$$v(x) = v_0 \exp(\gamma_H x^{\nu}) \sin\alpha, \quad \gamma_H = \frac{u_0^2}{2\nu_0 \omega_H^0}\left(l - 2\frac{\delta_H}{\omega_H^0}\right), \quad \nu = \frac{h}{z_0}.$$

(h is the height of uniform ionosphere, z_0 is the height of uniform acoustical atmosphere).

The solution of the inhomogenous Eq.(11), determining the strength of the electric field transformed by ionosphere's plasma waves, has the form

$$E(\varepsilon, \gamma_H, x) = C_1 I_0(2\varepsilon\sqrt{x}) + C_2 N_0(2\varepsilon\sqrt{x}) +$$
$$+ 2\pi v_0 \sin\alpha\beta C \left[N_0(2\varepsilon\sqrt{x}) \int t^2 I_0(2\varepsilon t)\, e^{\gamma_H t^{2\nu}}\, dt -\right.$$
$$\left. - I_0(2\varepsilon\sqrt{x}) \int t^2 N_0(2\varepsilon t)\, e^{\gamma_H t^{2\nu}}\, dt\right]. \qquad (12)$$

We shall formulate the boundary conditions as follows: at great values of z (at high altitudes) the solution must be limited, i. e. $E(\varepsilon, \gamma_H, x)$ must transform into a plane wave $\sim E_0 \exp(ikR)$, where k is the wave number ($\sim \omega/V$), R is the outer boundary of the F_2-layer.

The second boundary condition must express the absence of currents on the lower boundary of the ionosphere and atmosphere, in other words, the tangential component of the electric field at $\xi = \xi_*$ must vanish, $E|_{x = e^{-1}} = 0$. This condition serves for the determination of the spectrum ω_H of natural frequencies of ionosphere's free oscillations.

Making use of solution's limitation for $\xi \gg 1$ ($x \ll 1$), on the stength of the divergence of $N_0(2\varepsilon t)$ as $t \to 0$, coefficient's c_2 vanishing must be required. The terms in square brackets in (10) are restricted as $t \to 0$. In reality, the terms

$$N_0(2\varepsilon t)\int t^2 I_0(2\varepsilon t)\, e^{\gamma_H t^{2\nu}}\, dt, \quad I_0(2\varepsilon t)\int t^2 N_0(2\varepsilon t)\, e^{\gamma_H t^{2\nu}}\, dt$$

have near zero a value $\sim t^3 \log t$, i. e., they vanish, while $I_0(2\varepsilon t)|_{t\to 0} \sim 1$. Determining the outer boundary of the F_2-layer by means of R, known from the experiment, so that the condition $x_{outer} \ll 1$ is satisfied, we determine c_1. Inasmuch as the terms in square brackets vanish more rapidly than $\underline{x}$, we have in linear approximation by convergence powers x^2:

$$c_1 = E_0 \exp\left(i\frac{\omega}{V} \right).$$

Therefore, the general solution for the horiz l component of the electric field of forced F_2-layer's oscillations has t m

$$E(\varepsilon, \gamma_H, x) = E_0 e^{i\frac{\omega}{V}} \qquad \sqrt{x}) +$$
$$+ 2\pi r_0 \sin\varphi\, \beta C\, [N_0(2\varepsilon \sqrt{x})\int t^2 I_0(2\varepsilon t)\, e^{\gamma_H t^{2\nu}}\, dt - I_0(2\varepsilon \sqrt{x})\int t^2 N_0(2\varepsilon t)\, e^{\gamma_H t^{2\nu}}\, dt]. \quad (13)$$

Following are the denotations introduced: V is the phase velocity of a plane electrogmagnetic wave

$$\beta = \frac{\gamma_{i0}}{\omega_{iH}}, \quad \nu = \frac{h}{z_0} \quad \gamma_H = \frac{u_0^2}{2\nu_0 \omega_H^{\,\circ}}\left(i - 2\frac{\delta_H}{\omega_H^{\,\circ}}\right),$$

$$C = \frac{\Omega^2 \omega^2 \omega_{iH} h^2}{c^2 e}\left[\frac{1}{\omega_{iH} - \omega} + \frac{\gamma_{e0}}{\gamma_{i0}}\frac{1}{(\omega_{eH} + \omega)}\right],$$

$$\varepsilon = \frac{h}{c}\sqrt{\omega}\sqrt{\frac{\Omega^2}{\omega_{iH} - \omega} + \frac{\omega_p^2}{\omega_{eH} - \omega}} \times$$

$$\times \left\{ i - i\beta\omega_{iH}\, \frac{\dfrac{\Omega^2(\omega_{eH} - \omega)}{\omega_{iH}^2 - \omega^2} + \dfrac{\gamma_{ie}}{\gamma_{i0}}\dfrac{\omega_p^2(\omega_{iH} - \omega)}{\omega_{eH}^2 - \omega^2}}{\Omega^2(\omega_{eH} - \omega) + \omega_p^2(\omega_{iH} - \omega)} \right\} \quad (14)$$

Determination of the Spectrum of Natural Frequency of Ionosphere Oscillations. The solution of the inhomogenous field equation is expressed in the form of linear combination of cylindrical functions of zero order. The conversion of the tangential component of the electric field on the lower boundary of the ionosphere and atmosphere is the condition for the determination of natural frequencies at $\varepsilon_* = 1/3$. From (13) follows directly the equation

$$I_0\left(\frac{2}{\sqrt{3}}\varepsilon\right) = 0.$$

Let us estimate the value of β applicably to the F2-layer. For a concentration of electrons and ions $\sim 10^6$ cm^{-3}, we have

$$\Omega = 1.5\cdot 10^6\ sec^{-1};\quad \omega_{iH} = 3.5\cdot 10^4;\quad \omega_p = 7.5\cdot 10^7\ sec^{-1};\ \omega_{eH} = 1.7\cdot 10^6 s^{-1}.$$

The roots of the Bessel function of zero order are well known with great precision. For greater values of the argument that may take place in our case, the roots of $I_0(\varepsilon_m)$ are given by the expansion

$$\varepsilon_m \approx \frac{\pi}{4}(4m-1) + \frac{1}{2\pi(4m-1)} - \frac{31}{6\pi^3(4m-1)^3} + \frac{3779}{5\pi^5(4m-1)^5} - \cdots$$

This series is valid for the computation of all roots (with the exception of the least ε_m, m = 1.2) with a precision of at least five signs. In order to determine the natural frequencies it is necessary to resolve the expression with respect to ω_0. The equation is complex and cumbersome; however, the presence of the smallness parameter β allows us to seek the solution in the form of expansion in series by powers of the smallness parameter. Omitting the intermediate calculations, we find $\omega_m^{(0)}$ that is, the approximation when the collisions of charged particles with neutral ones may be neglected :

$$\omega_m^{(0)} = \omega_{iH}\left\{1 + \left(\frac{c}{h}\frac{\varepsilon_m}{\omega_p}\sqrt{\frac{M}{2m}}\right)^2 \mp \left[1 + \left(\frac{c}{h}\frac{\varepsilon_m}{\omega_p}\sqrt{\frac{M}{2m}}\right)^4\right]^{1/2}\right\}.$$

Taking into account the collisions, we determine in linear approximation the correction to $\omega_m^{(0)}$, introduced by the attenuation of electric fields in the ionosphere at the expense of collisions

$$\omega_m^{(1)} = i\beta\omega_m^{(0)2}\left(\frac{\Omega^2}{\omega_{iH} - \omega_m^{(0)}} + \frac{\omega_p^2}{\omega_{eH} - \omega_m^{(0)}}\right) \times \frac{\omega_{iH}^2\omega_{eH}\left[1 + \frac{\gamma_{e0}}{\gamma_{i0}}\left(\frac{m_0}{M}\right)^2\right]}{(2\omega_{iH} - \omega_m^{(0)})^2(\Omega^2\omega_{eH} - \omega_p^2\omega_{iH})}$$

In the case when one of the acoustic wave frequencies is found to be close in value to one of the natural frequencies of plasma oscillations, resonance is possible. Inasmuch as in the expression for the electric fields (13) resonance terms enter in the form of integrals, the resonance increase will have an implicit character, while the resonance peak will be blurred. The presence of resonances is linked with the phenomenon of plasma wave reflection from regions with increasing electron concentration. Note that on the general curve of function $E(\varepsilon_m, \gamma_n, x)$ resonance peaks will be concentrating as the number m increases, while the amplitude will decrease. This problem was resolved at the outset in the linear approximation and this is why the resonance accretion of elecric field amplitudes limits the multiplier proportional to parameter β.

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